AdS/Ricci-flat correspondence and the Gregory-Laflamme instability

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We show that for every asymptotically AdS solution compactified on a torus there is a corresponding Ricci-flat solution obtained by replacing the torus by a sphere, performing a Weyl rescaling of the metric and appropriately analytically continuing the dimension of the torus/sphere (as in generalized dimensional reduction). This correspondence should allow us to develop a holographic dictionary for Ricci-flat spacetimes. In particular, it maps Minkowski spacetime to AdS on a torus, the holographic stress energy tensor of AdS to the stress energy tensor due to a brane localized in the interior of spacetime and AdS black branes to (asymptotically flat) Schwarzschild black branes. Applying it to the known solutions describing the hydrodynamic regime in AdS/CFT, we compute the dispersion relation of the Gregory-Laflamme unstable modes through cubic order in the wavenumber, finding remarkable agreement with numerical data. We further obtain the fluid dual to Rindler spacetime and show that its transport coefficients through second order follow from the AdS ones.

Introduction. Holography is very well understood when the spacetime is asymptotically (locally) Anti-de Sitter (AdS). However, the general arguments for holography based on black hole physics are insensitive to asymptotics, suggesting that one should be able to develop a holographic dictionary for spacetimes with non-AdS asymptotics. In some cases, a string theoretic construction implies that a spacetime which is not asymptotically AdS should admit a holographic description. Indeed, the original decoupling argument [1] also extends to nonconformal branes [2] and these give rise to spacetimes that are not asymptotically AdS. Nevertheless, a detailed holographic dictionary can be set up for such spacetimes, [3, 4].

It turns out that these cases can be linked to the asymptotically AdS case via a generalized dimensional reduction [5]: one starts from asymptotically AdS spacetimes in higher dimensions, dimensionally reduces on a torus and then analytically continues the dimension of the torus (which only appears as a parameter in the reduced theory) to a real number. This procedure commutes with all operations needed to establish a precise holographic dictionary: one can obtain the general asymptotic solutions, the covariant boundary counterterms and the holographic 1-point functions of the reduced theory from the corresponding results of the parent AdS theory (see also [6] for the finite density case).

In this Letter we will apply the same method in order to link Ricci-flat solutions to asymptotically AdS spacetimes. This then provides the starting point for developing a detailed holographic dictionary for Ricci-flat spacetimes, which we will discuss in detail elsewhere [7]. Here we will present the main construction and one application, namely the analytic study of the Gregory-Laflamme (GL) instability [8, 9]. The GL instability is an instability of black strings to linearized gravitational perturbations that break their translational symmetry but preserve their transverse spherical symmetry. The linearized equations were originally solved numerically and one of the main results is the dispersion relation of the unstable mode. Since then there has been much progress in the nonlinear evolution of the perturbation (see e.g. [9, 10]). Here we will use the connection to AdS to obtain an analytic long-wavelength approximation of the dispersion relation. Our results reproduce the quadratic approximation of the dispersion relation found in [11], extend it to one order higher and lead to a striking agreement between the analytic expression and the numerical results.

AdS/Ricci-flat correspondence. Consider pure AdS gravity in $(d + 1)$ dimensions with cosmological constant $\Lambda = -d(d-1)/2\ell^2$ and compactify $(d - p - 1)$ spatial dimensions on a diagonal torus of volume $vol(T)$. Let us label the coordinates as $x^M = \{\rho, z^\mu\}, (\mu = 0, ..., d-1)$ and $z^\mu = \{x^a, \vec{y}\}, (a = 0, ..., p)$, with $\vec{y}$ the torus coordinates. Then corresponding to any solution of the form,

$$\mathrm{d}s_\Lambda^2 = \mathrm{d}\hat{s}_{p+2}^2(\rho, x) + e^{2\phi(\rho,x)/(d-p-1)}\mathrm{d}\vec{y}^2, \qquad (1)$$

there is a $D = n + p + 3$ Ricci-flat solution given by

$$\mathrm{d}s_0^2 = e^{2\phi(\rho,x)/(n+p+1)} \left(\mathrm{d}\hat{s}_{p+2}^2(\rho, x) + \ell^2\mathrm{d}\Omega_{n+1}^2 \right), \quad (2)$$

where $d\Omega_{n+1}^2$ is the metric of the unit round $(n+1)$-sphere of volume Ω_{n+1} and d and n are related by

$$n \leftrightarrow -d. \tag{3}$$

In other words, if we are given a solution (1) then the correspondence instructs us to extract from it the $(p+2)$-metric $\hat{g}$ and the scalar field ϕ, replace in them all explicit factors of d by $-n$ and then insert them in (2). This metric is then Ricci-flat.[1] Similarly, given (2) one can follow the same steps (now with n replaced with $-d$) in order to obtain the Einstein metric (1). Note that this correspondence requires knowing the solutions for general n and d. Furthermore, the AdS radius ℓ is mapped to the radius of the sphere in (2).

This correspondence is proven by substituting the Ansätze (1) and (2) in their corresponding field equations, expressing them as equations for the $(p+2)$-dimensional metric $\hat{g}$ and the scalar field ϕ and showing the equations map to each other if we use (3). This is another instance of generalized dimensional reduction: reducing AdS gravity on the torus to $(p+2)$ dimensions, d now appears as a parameter which can be continued to negative values. This theory then matches the one obtained by starting with Einstein gravity without cosmological constant and reducing on a sphere to $(p+2)$ dimensions.

This correspondence implies that this class of Ricci-flat solutions has an underlying holographic structure, namely that inherited from the AdS ones. Asymptotically locally AdS spacetimes come equipped with a conformal structure represented by an arbitrary metric on the boundary. This metric in AdS/CFT acts as the source for the dual stress energy tensor. Using the Ricci-flat/AdS correspondence, we find that this class of Ricci-flat solutions will also depend on an arbitrary metric and one may follow the usual steps to set up a holographic dictionary for Ricci-flat solutions. This is highly nontrivial as the naive extension of the AdS holographic computations to Ricci-flat solutions reveals that the asymptotia depend on constrained data and the infinities in the on-shell action are nonlocal with respect to them [13, 14]. The derivation of the map (following closely [6]), a number of generalizations and the full holographic analysis will be presented in a series of papers [7]. Here we will restrict ourselves to flat boundary metrics.

The simplest case to analyze is AdS spacetime with $(d-p-1)$ of the boundary directions compactified on a torus,

$$ds_\Lambda^2 = \frac{\ell^2}{r^2}\left(dr^2 + \eta_{ab}dx^a dx^b + d\vec{y}^2\right), \tag{4}$$

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[1] A similar, more restricitive correspondence was noted earlier in [12].

where $\rho = r^2$. Matching with (1) we find $\phi(r,x) = (p+1-d)\ln r/\ell$ and after applying (3) and substituting in (2) we obtain

$$ds_0^2 = (dr^2 + r^2 d\Omega_{n+1}^2) + \eta_{ab}dx^a dx^b, \tag{5}$$

which is simply D-dimensional Minkowski spacetime. On the AdS side, we fixed η_{ab} as the boundary condition for the metric on the noncompact boundary coordinates. This maps on the Ricci-flat side to the metric on a p-brane located at $r = 0$, the origin in transverse space. The radial direction in AdS is the transverse distance from the p-brane. This is reminiscent of the way AdS spacetime emerges from D/M-brane spacetimes in the decoupling limit [1].

Let us now add excitations on top of AdS. The general asymptotic solution in Fefferman-Graham (FG) gauge with a flat boundary metric is given by [15],

$$ds_\Lambda^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}\left(\eta_{\mu\nu} + \rho^{d/2}g_{(d)\mu\nu} + \cdots\right)dz^\mu dz^\nu, \tag{6}$$

where here and throughout the rest of this Letter we set the AdS radius $\ell = 1$. $g_{(d)\mu\nu}$ is related to the expectation value of the dual stress energy tensor, [15],

$$T_{\mu\nu} = \frac{d}{16\pi G_N}g_{(d)\mu\nu}, \tag{7}$$

which satisfies (as a consequence of the gravitational field equations) [15, 16],

$$\partial^\mu T_{\mu\nu} = 0, \qquad T_\mu{}^\mu = 0. \tag{8}$$

These are the correct Ward identities for the CFT on a flat background.

Compactifying on a $(d-p-1)$ torus, the metric $\hat{g}$ and the scalar field ϕ can be extracted from (1):

$$d\hat{s}^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}\left(\eta_{ab} + \rho^{d/2}(\hat{g}_{(d)ab} + \rho\hat{g}_{(d+2)ab} + \ldots)\right)dx^a dx^b,$$
$$\phi = \frac{(p+1-d)}{2}\ln\rho + \rho^{d/2}\hat{\phi}_{(d)} + \rho^{d/2+1}\hat{\phi}_{(d+2)} + \ldots, \tag{9}$$

where the displayed coefficients are related to the expectation values of the stress energy tensor $\hat{T}_{ab}$ and of the scalar operator $\mathcal{O}_\phi$ of the $(p+2)$-dimensional theory [5]:

$$\hat{T}_{ab} = \frac{d}{16\pi\hat{G}_N}\hat{g}_{(d)ab}, \quad \mathcal{O}_\phi = -\frac{d(d-p-1)}{32\pi\hat{G}_N}\hat{\phi}_{(d)}, \tag{10}$$

while $\hat{g}_{(d+2)ab} = -\Box\hat{g}_{(d)ab}/(2d(d+2))$ and $\hat{\phi}_{(d+2)} = -\Box\hat{\phi}_{(d)}/(2d(d+2))$. $\hat{G}_N = G_N/vol(T)$ is the $(p+2)$-dimensional Newton's constant. The stress energy tensor satisfies the expected trace and diffeomorphism Ward identities

$$\partial^a\hat{T}_{ab} = 0, \qquad \hat{T}_a{}^a = (d-p-1)\mathcal{O}_\phi. \tag{11}$$

We can now apply the AdS/Ricci-flat correspondence to obtain the corresponding Ricci-flat solution:

$$ds_0^2 = (\eta_{AB} + h_{AB} + \dots)dx^A dx^B = \left(1 - \frac{16\pi \hat{G}_N}{n\, r^n}(1+\right.$$

$$\left.\frac{r^2}{2(n-2)}\Box)\hat{\mathcal{O}}_\phi(x)\right)\left(dr^2 + \eta_{ab}dx^a dx^b + r^2 d\Omega_{n+1}^2\right)$$

$$-\frac{16\pi \hat{G}_N}{n\, r^n}(1 + \frac{r^2}{2(n-2)}\Box)\hat{T}_{ab}(x)dx^a dx^b + \dots \quad (12)$$

where $\rho = r^2$ and x^A are D-dimensional coordinates. Defining $\bar{h}_{AB} = h_{AB} - \frac{h^C_{\ C}}{2}\eta_{AB}$,

$$\Box \bar{h}_{AB} = 16\pi \hat{G}_N \Omega_{n+1}\delta_A^{\ a}\delta_B^{\ b}\hat{T}_{ab}\delta^{n+2}(r) \quad (13)$$

follows through second order terms in (boundary) derivatives. Comparing with the linearized Einstein equations we conclude that the holographic stress energy tensor takes a new meaning: it is (proportional to) minus the stress energy tensor due to a p-brane located at $r = 0$ that sources the linearized gravitational field h_{AB}.

Let us now consider how black objects are mapped to each other under the AdS/Ricci-flat map. The planar AdS black brane reads,

$$ds_\Lambda^2 = z^2(-f(z)d\tau^2 + d\vec{x}^2 + d\vec{y}^2) + \frac{dz^2}{z^2 f(z)}, \quad (14)$$

where $x^a = \{\tau, \vec{x}\}$, the $\vec{y}$ coordinates parametrize a torus, as before, and $f(z) = 1 - 1/(bz)^d$. Applying the map and changing coordinates, $z = 1/r$, we obtain

$$ds_0^2 = -f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{n+1}^2 + d\vec{x}^2, \quad (15)$$

where $f(r) = 1 - (b/r)^n$, i.e. the Schwarzschild black p-brane.

Gregory-Laflamme instability. Black strings and branes suffer from a linearized s-wave instability [8]. Most of our results are applicable for p-branes but for concreteness we will focus on the black string case, $p = 1$. Since the perturbations respect the transverse spherical symmetry, the perturbed metrics fall within the class of metrics (2). This means that we can use the AdS/Ricci-flat correspondence to map them to linear perturbations of the AdS black brane (14). These perturbations are known analytically for small frequencies and we will use this result to obtain an approximate analytic formula for the dispersion relation of the GL mode.

The GL instability is a long wavelength instability: only modes with wavelength greater than a critical value are unstable and the critical value is related to the thickness of the brane b. Thus, compactifying the x coordinate on a small enough circle may stabilize the black string. Here we consider the case x is noncompact. In this case, unstable modes appear below a critical wavenumber k_c.

Their (imaginary) frequency Ω and momentum k satisfy a dispersion relation $\Omega(k)$, which we would like to compute.

We will analyze the problem in the small k limit. This regime is mapped under the AdS/Ricci-flat correspondence to the gravitational solutions describing the hydrodynamic regime of the dual CFT. These solutions were obtained in [9, 17, 18] by solving the full (nonlinear) Einstein equations in a (boundary) derivative expansion. The AdS/Ricci-flat correspondence then leads to the construction of similar nonlinear perturbations of the black string, which may shed light on the nonlinear GL instability. These (nonlinear) solutions to second order in gradients will be presented in [7]. Here we will linearize in the amplitude of perturbations and discuss their dispersion relation.

The gravitational solution is essentially encoded on the dual stress energy tensor $T_{\mu\nu}$. On the AdS side, this is the statement of holography. On the Ricci-flat side, we saw that $\hat{T}_{ab}$ sources the linearized gravitational field. Thus, to determine the dispersion relation, it suffices to solve the Ward identities (8). This provides also a direct link with the blackfold formalism [9, 19–21], which was already used in order to study the GL instability [11].

Consider first the equilibrium solutions. On the AdS side, the solution is dual to an ideal conformal fluid. The equation of state follows from (8)

$$\varepsilon = (d-1)P \quad (16)$$

where ε and P are the energy and pressure densities. Using the AdS/Ricci-flat map one finds,

$$\tilde{\varepsilon} = -(n+1)\tilde{P}, \qquad c_s^2 = \frac{\partial \tilde{P}}{\partial \tilde{\varepsilon}} = -\frac{1}{n+1}, \quad (17)$$

where $\tilde{\varepsilon}$ and $\tilde{P}$ are the ADM energy and pressure densities. Since the speed of sound c_s is imaginary, there is an instability for the sound modes which is linked to the GL instability [20].

The holographic stress energy tensor for gravitational solutions describing the hydrodynamic regime has been obtained for general d and to second order gradients in [18]. Defining the d-velocity $u^\mu = \{u^a, \vec{0}\}$ with components only in the noncompact directions ($u^\mu u_\mu = -1$), the projection operator $P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$ orthogonal to u_μ and the shear tensor $\sigma_{\mu\nu} = P_\mu^\kappa P_\nu^\lambda \partial_{(\kappa}u_{\lambda)} - P_{\mu\nu}\partial \cdot u/(d-1)$, the terms contributing to the linearized analysis are

$$T_{\mu\nu} = P(\eta_{\mu\nu} + d u_\mu u_\nu) - 2\eta\sigma_{\mu\nu}$$
$$- 2\eta(\tau_\omega - b)P_\mu^{\ \kappa}P_\nu^{\ \lambda}u \cdot \partial\sigma_{\kappa\lambda} \quad (18)$$

with

$$P = \frac{b^{-d}}{16\pi G_N}, \quad \eta = \frac{b^{1-d}}{16\pi G_N}, \quad \tau_\omega = -\frac{b}{d}H_{2/d-1}, \quad (19)$$

giving the pressure, shear viscosity η and second order coefficient τ_ω. H_z is the Harmonic number function. Note

that H_z is an analytic function in z, defined over the whole complex z-plane with singular points at all negative integers.

For any solution of the relativistic Navier-Stokes equation, $\partial_\mu T^{\mu\nu} = 0$, with $T_{\mu\nu}$ as above, there is a corresponding gravitational solution. To obtain the dispersion relation for the sound mode we consider an infinitesimal variation of the velocity in the x direction, $u^x \sim \delta u^x \exp(-i\omega t + ikx)$ and of the energy density $\varepsilon \sim \varepsilon_0 + \delta\varepsilon \exp(-i\omega t + ikx)$, where ε_0 is the equilibrium energy density. Applying the AdS/Ricci-flat correspondence at the level of solutions and following [22] leads to the dispersion relation for the sound mode of the black string to order k^3. Defining $\Omega = -i\omega$ we find[2]

$$\Omega = \frac{1}{\sqrt{n+1}}k - \frac{(2+n)}{n(1+n)}k^2 + \frac{(2+n)[2+n(2\tilde{\tau}_\omega - 1)]}{2n^2(1+n)^{3/2}}k^3, \quad (20)$$

where $\tilde{\tau}_\omega = (1/n)H_{-2/n-1}$ and we have set $b = 1$. The first two terms reproduce the quadratic approximation in [11].

As mentioned above, H_z is well-defined when its index is a nonnegative integer, so the dispersion relation (20) is valid for all cases, except when $n = 1, 2$,[3] corresponding to five and six-dimensional black strings. These cases are subtle because under (3) they map to $d = -1, -2$, and terms in the FG expansion (6) that are normally distinct now become degenerate: for example, when $d = -2$, the pairs $\{g_{(0)}, g_{(d+2)}\}$ and $\{g_{(d)}, g_{(2d+2)}\}$. There are similar degeneracies for any integer n, but this subtlety becomes relevant only at specific order in the derivative expansion, which is n ($n+1$), if n is even (odd). Thus, if one works to second order in derivatives only the $n = 1, 2$ cases have to be analyzed separately. The complete analysis (see [7]) leads to

$$n = 1: \quad \Omega = \frac{1}{\sqrt{2}}k - \frac{3}{2}k^2 + \frac{75}{16\sqrt{2}}k^3 + O(k^4), \quad (21)$$

$$n = 2: \quad \Omega = \frac{1}{\sqrt{3}}k - \frac{2}{3}k^2 + \frac{5}{6\sqrt{3}}k^3 + O(k^4). \quad (22)$$

To obtain (20) we assumed that k is small and in general one should not expect that it accurately captures the entire range of unstable modes, $k \leq k_c$. Nevertheless, the dispersion relation (20) agrees remarkably well with the numerical data. In Figure 1 we plot (20) against the numerical data [23] for two cases $n = 7$ and $n = 100$ ($\tilde{k}$ and $\tilde{\Omega}$ are defined below in (23)). When $n = 100$, the agreement all the way up to the threshold mode is striking. In the same graph we also include the quadratic
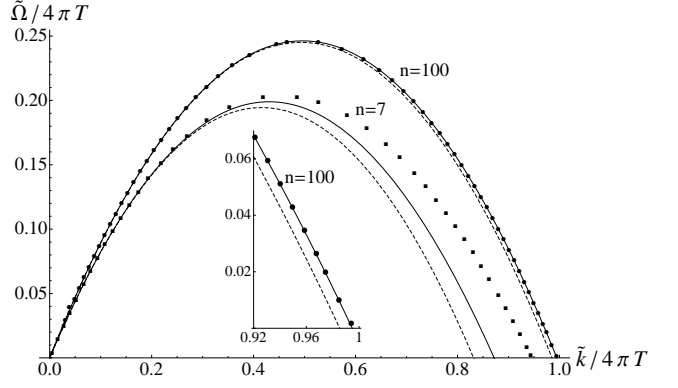

Figure 1. Gregory-Laflamme dispersion relation for $n = 7$ and $n = 100$. The dots/squares represent numerical data for $n = 100/n = 7$, the solid line the cubic approximation and the dashed line the quadratic approximation. In the insert we zoom into the region close to the threshold mode ($n = 100$).

approximation to illustrate the improvement due to the k^3 term.

As it is clear from the $n = 7$ case, for low n the agreement becomes less and less good as k becomes larger and larger. For $n = 7$, (20) estimates the largest Ω with an error of 2%, and the intercept k_c is off by 9%. For $n = 100$, the discrepancy is 0.01% at the maximum and 0.02% at the intercept. For very low n ($n < 5$) the quadratic approximation is actually better than the cubic one when k is large enough, indicating that (20) is actually an asymptotic series rather than a convergent one. When $n = 1, 2$ the cubic approximation no longer captures the correct finite k behavior – there is no threshold mode.

To understand better the large n behavior,[4] let us zoom in this region by scaling the frequency and wavenumber as [11],

$$\tilde{\Omega} = n\Omega, \qquad \tilde{k} = \sqrt{n}\,k, \quad (23)$$

while keeping the temperature, $T = n/(4\pi b)$, fixed. The relative scaling of Ω to k follows from the fact that the speed of sound goes to zero as $1/\sqrt{n}$ in this limit, and the overall scaling is obtained by requiring that the dispersion relation has a nontrivial limit. In terms of these variables the dispersion relation, truncated at $\tilde{k}^3$, takes the following suggestive form

$$\tilde{\Omega} = \tilde{k}\left(1 - \frac{\tilde{k}}{4\pi T}\right) - \frac{\tilde{k}}{n}\left(\frac{1}{2} + \frac{\tilde{k}}{4\pi T} - \frac{\tilde{k}^2}{(4\pi T)^2}\right) + O\left(\frac{1}{n^2}\right). \quad (24)$$

Solving for the threshold mode from (24), we find that $\tilde{k}_c = 4\pi T(1 - 1/(2n) + O(1/n^2))$, in precise agreement

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[2] Note that the naive application of the map directly in the dispersion relation of [22] would lead to incorrect sign of the k^2 term. This and other such subtlties will be explained in [7].

[3] For $n \to \infty$, $H_{-2/n-1}$ is singular but $\tilde{\tau}_\omega \to 1/2$.

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[4] The GL instability in a large number of dimensions was first studied in [24].

with [25]. This means that $k_c/T \sim 4\pi/\sqrt{n} \to 0$, as $n \to \infty$, and therefore the threshold mode falls within the long-wavelength approximation.

The fluid dual to vacuum Einstein gravity. In the special case $n = -1$,[5] we are describing hydrodynamic perturbations of Rindler spacetime, whose fluid dual was studied in [26–29]. In particular, this fluid has zero equilibrium energy density $\tilde{\varepsilon}$ and it is characterized by its pressure density $\tilde{P}$. Indeed, as $n \to -1$ with $\tilde{P}$ fixed we find that (17) yields $\tilde{\varepsilon} = 0$. All transport coefficients through second order may be obtained by reducing the results of [18] to $p + 2$ dimensions (as described earlier around (18) but without linearizing) and converting to the isotropic gauge used in [28]. Taking the limit $n \to -1$ leads to exact agreement with [27–29].

Conclusions. We presented a map between asymptotically AdS spacetimes and Ricci-flat solutions. This map should allow us to develop a detailed holographic dictionary for Ricci-flat spacetimes. To do this, we would need to allow for arbitrary source terms on the AdS side (such terms were set to zero here). The map also implies that there is an inherited (hidden) conformal invariance on the Ricci-flat side. For example, this generalized conformal structure [4] dictates the form of the transport coefficients for nonconformal branes [5] and related systems [6] and (using the results in this Letter) also explains the form of the first order transport coefficient in [11]. In recent black hole literature a hidden conformal invariance was found in a number of instances, see, for example, [30–33]. It would be interesting to understand if there is a link between these works and the conformal symmetry inherited from AdS via the AdS/Ricci-flat map. We intend to pursue these and related questions in the near future.

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[1] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[2] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, "Supergravity and the large N limit of theories with sixteen supercharges," Phys. Rev. D **58**, 046004 (1998) [arXiv:hep-th/9802042].

[3] T. Wiseman and B. Withers, "Holographic renormalization for coincident Dp-branes," JHEP **0810**, 037 (2008) [arXiv:0807.0755][hep-th].

[4] I. Kanitscheider, K. Skenderis and M. Taylor, "Precision holography for non-conformal branes," JHEP **0809**, 094 (2008) [arXiv:0807.3324][hep-th].

[5] I. Kanitscheider and K. Skenderis, "Universal hydrodynamics of non-conformal branes," JHEP **0904** (2009) 062 [arXiv:0901.1487][hep-th].

[6] B. Goutéraux, J. Smolic, M. Smolic, K. Skenderis and M. Taylor, "Holography for Einstein-Maxwell-dilaton theories from generalized dimensional reduction," JHEP **1201** (2012) 089 [arXiv:1110.2320][hep-th].

[7] M. M. Caldarelli, J. Camps, B. Goutéraux and K. Skenderis, to appear.

[8] R. Gregory and R. Laflamme, "Black strings and p-branes are unstable," Phys. Rev. Lett. **70** (1993) 2837 [arXiv:hep-th/9301052].

[9] For a recent review see the relevant chapter in *Black Holes in Higher Dimensions*, Edited by: Gary T. Horowitz, Cambridge University Press, 2012.

[10] L. Lehner and F. Pretorius, "Black Strings, Low Viscosity Fluids, and Violation of Cosmic Censorship," Phys. Rev. Lett. **105** (2010) 101102 [arXiv:1006.5960][hep-th].

[11] J. Camps, R. Emparan and N. Haddad, "Black Brane Viscosity and the Gregory-Laflamme Instability," JHEP **1005** (2010) 042 [arXiv:1003.3636][hep-th].

[12] C. Charmousis and R. Gregory, "Axisymmetric metrics in arbitrary dimensions," Class. Quant. Grav. **21** (2004) 527 [arXiv:gr-qc/0306069].

[13] S. de Haro, K. Skenderis and S. N. Solodukhin, "Gravity in warped compactifications and the holographic stress tensor," Class. Quant. Grav. **18**, 3171 (2001) [arXiv:hep-th/0011230].

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[5] When $n \to -1$, the sphere collapses in (15): changing coordinates to $(t_R = t/b + \log f(r),\ r_R = b^2 f(r))$, the metric becomes Minkowski spacetime in ingoing Rindler coordinates.

[14] K. Skenderis, "Lecture notes on holographic renormalization," Class. Quant. Grav. **19** (2002) 5849 [arXiv:hep-th/0209067].

[15] S. de Haro, S. N. Solodukhin and K. Skenderis, "Holographic reconstruction of space-time and renormalization in the AdS / CFT correspondence," Commun. Math. Phys. **217**, 595 (2001) [arXiv:hep-th/0002230].

[16] M. Henningson and K. Skenderis, "The Holographic Weyl anomaly," JHEP **9807** (1998) 023 [arXiv:hep-th/9806087].

[17] S. Bhattacharyya, V. E. Hubeny, S. Minwalla and M. Rangamani, "Nonlinear Fluid Dynamics from Gravity," JHEP **0802** (2008) 045 [arXiv:0712.2456][hep-th].

[18] S. Bhattacharyya, R. Loganayagam, I. Mandal, S. Minwalla and A. Sharma, "Conformal Nonlinear Fluid Dynamics from Gravity in Arbitrary Dimensions," JHEP **0812** (2008) 116 [arXiv:0809.4272][hep-th].

[19] R. Emparan, T. Harmark, V. Niarchos and N. A. Obers, "World-Volume Effective Theory for Higher-Dimensional Black Holes," Phys. Rev. Lett. **102** (2009) 191301 [arXiv:0902.0427][hep-th].

[20] R. Emparan, T. Harmark, V. Niarchos and N. A. Obers, "Essentials of Blackfold Dynamics," JHEP **1003** (2010) 063 [arXiv:0910.1601][hep-th].

[21] J. Camps and R. Emparan, "Derivation of the blackfold effective theory," JHEP **1203** (2012) 038 [Erratum-ibid. **1206** (2012) 155] [arXiv:1201.3506][hep-th].

[22] R. Baier, P. Romatschke, D. T. Son, A. O. Starinets and M. A. Stephanov, "Relativistic viscous hydrodynamics, conformal invariance, and holography," JHEP **0804** (2008) 100 [arXiv:0712.2451][hep-th].

[23] P. Figueras, private communication.

[24] B. Kol and E. Sorkin, "On black-brane instability in an arbitrary dimension," Class. Quant. Grav. **21** (2004) 4793 [arXiv:gr-qc/0407058].

[25] V. Asnin, D. Gorbonos, S. Hadar, B. Kol, M. Levi and U. Miyamoto, "High and Low Dimensions in The Black Hole Negative Mode," Class. Quant. Grav. **24** (2007) 5527 [arXiv:0706.1555][hep-th].

[26] I. Bredberg, C. Keeler, V. Lysov and A. Strominger, "From Navier-Stokes To Einstein," JHEP **1207**, 146 (2012) [arXiv:1101.2451][hep-th].

[27] G. Compere, P. McFadden, K. Skenderis and M. Taylor, "The Holographic fluid dual to vacuum Einstein gravity," JHEP **1107**, 050 (2011) [arXiv:1103.3022][hep-th].

[28] G. Compere, P. McFadden, K. Skenderis and M. Taylor, "The relativistic fluid dual to vacuum Einstein gravity," JHEP **1203**, 076 (2012) [arXiv:1201.2678][hep-th].

[29] C. Eling, A. Meyer and Y. Oz, "The Relativistic Rindler Hydrodynamics," JHEP **1205**, 116 (2012) [arXiv:1201.2705][hep-th].

[30] A. Castro, A. Maloney and A. Strominger, "Hidden Conformal Symmetry of the Kerr Black Hole," Phys. Rev. D **82**, 024008 (2010) [arXiv:1004.0996][hep-th].

[31] S. Bertini, S. L. Cacciatori and D. Klemm, "Conformal structure of the Schwarzschild black hole," Phys. Rev. D **85**, 064018 (2012) [arXiv:1106.0999][hep-th].

[32] M. Cvetic and F. Larsen, "Conformal Symmetry for General Black Holes," JHEP **1202**, 122 (2012) [arXiv:1106.3341][hep-th].

[33] S. Carlip, "Effective Conformal Descriptions of Black Hole Entropy," Entropy **13**, 1355 (2011) [arXiv:1107.2678][gr-qc].